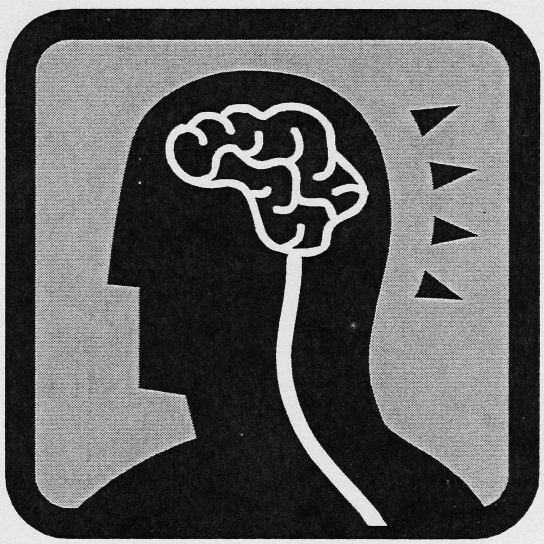
Notes for Educators



Strategies for Learning Addition Facts

Doubles

- List all combinations, either randomly or in order, and discuss patterns; emphasize that the two addends you add together are the same

- "Double it" drawings on grid paper – use grid paper and have students make a row of 6 objects. Then ask them to "double" it, making another row of 6 objects of the same size. Repeat several times with other numbers.

- Pictures of everyday doubles – post pictures of common doubles such as fingers on each hand (5 + 5), egg cartons (6 + 6), 6 packs of water bottles (3 + 3).

- Read the story, *Two of Everything,* by Lily Toy Hong (Whitman & Company, 1993) which involves a doubling pot. Draw a doubling pot on the board or overhead and place a number in it. Have students mentally double numbers.

Doubles + 1

- Show strategy with manipulatives - physically form doubles

x x x x x x 6
x x x x x x x +7

6 + 6 = 12 12 + 1 = 13

Doubles + 2 or Sharing Facts

- Show strategy with manipulatives - physically form doubles. This works well for students who see one number as "2 MORE" than the other.

x x x x x x 6
x x x x x x x x +8

6 + 6 = 12 12 + 2 = 14

- Concept of BALANCE - show on a balance how if one side has two more, the extra two can be split so one side keeps one of the two (decreasing its total by 1) and the other side gets one (increasing its total by 1). Now both sides have the same number - they balance and form a doubles fact. This works well for students who see "a NUMBER MISSING BETWEEN" the two given numbers.

- Concept of FAIR SHARING - use context of sharing things, sharing in the same manner as the balance idea above.

Here's a picture of how sharing works:

6 x x x x x x
+8 x x x x x x x (x)
14

with sharing looks like: x x x x x x x 7
x x x x x x x +7
14

Adding 0 and Adding 1

- Use room posters to generalize effects of adding of 0 and 1. Have students state in their own words what happens when you add 0 or 1 to a number. E.g. *If you add 0 to a number you get the same number. If you add one to a number you get the next number you say in counting.*

Adding 10 and Adding 9

- Use ten and nine frames and manipulatives to show patterns (see page 29)

- Show multiple problems so students see the + 10 and +9 patterns and can determine the relationship of + 9 to + 10

- Think clouds

$$\begin{array}{r} 9 \\ +\ 6 \\ \hline 15 \end{array}$$

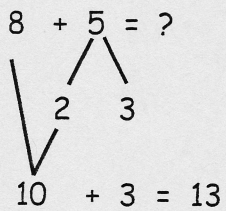
(think 10 + 6)

Using What You Know

- Counting on works well if one of the addends is a smaller number.

- Using 10 as a bridge

  Break apart (decompose) smaller number, get to 10, add on the rest. This can be shown using the connecting lines below, but eventually can be done mentally.

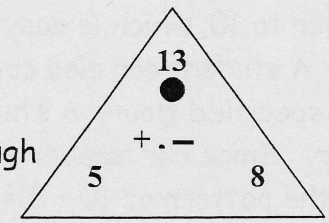


Algebraic Connections to Strategies

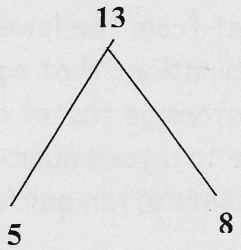
Doubles $x + x = y$ $2x = y$	Doubles + 1 $x + (x + 1) = y$ $x + x + 1 = y$ $2x + 1 = y$
Doubles + 2 $x + (x + 2) = y$ $x + x + 2 = y$ $2x + 2 = y$	Sharing $x + (x + 2) = y$ $x + (x + 1 + 1) = y$ $(x + 1) + (x + 1) = y$
Adding 9 $9 + x = y$ $(10 - 1) + x = y$ $(10 + x) - 1 = y$	Adding 0 $0 + x = x$ $x + 0 = x$

Strategies for Learning Subtraction Facts

- Think how addition and subtraction are related through
  - Fact families
    5 + 8 = 13
    8 + 5 = 13
    13 - 5 = 8
    13 - 8 = 5
  - Fact triangles
  - Math Mountains

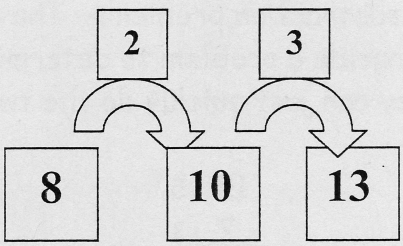


Fact Triangle



Math Mountain

- Use addition and 10 as a bridge
  - Think of the distance (difference) between the numbers
    - e.g. for 13 - 8 think of the distance from 8 to 13
    - go from 8 to 10 (difference of 2) and then from 10 to 13 (difference of 3) - put the parts together (2 + 3) to get 5 , the entire difference



Subtraction has multiple meanings. One is the idea of actually SUBTRACTING or TAKING AWAY one number from another. A second meaning is the DIFFERENCE between two numbers which is the same amount whether you count up from the "smaller" to the "larger" number or count back from the "larger" to the "smaller" number.

Students might be familiar with the DIFFERENCE concept of subtraction from riding an elevator. If you get on at one floor and go up or down to another floor, the distance you travel is the difference between the numbers of the two floors.

The RIDE THE EXPRESS ELEVATOR MODEL (see page 30) will help students see the difference between a lower floor number and a teen floor number in two parts. This elevator always makes a stop at the 10th floor. So, we just have to know how to get from the lower number to 10, which is easy if a student knows the combinations that equal 10. A student can also count up to the 10th floor. Some elevators go faster after a specified floor. A student can go quickly from the 10th floor to a teen number floor. Since our number system is a base ten system, students often quickly see the pattern of 10 + 5 = 15, 10 + 6 = 16, 10 + 7 = 17, etc.

To make the RIDE THE EXPRESS ELEVATOR MODEL cut the 2 number strips off the side and tape together to make a number strip from 1 to 18. Cut slits above and below the floor indicator on the elevator wide enough to allow the strip to move freely. Insert the strip from the back, across the floor indicator and push the end through the top slit to the back again.

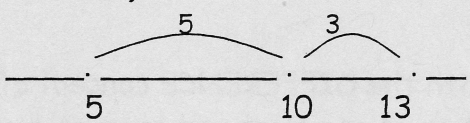
To do 15 - 7, set the elevator floor indicator to 7. Think 7 to 10 is 3 floors and then it's a quick 5 floors from 10 to 15. Put the two parts together (3 + 5) to make 8, the answer to 15 - 7. To do 16 - 9, it takes 1 floor to get to 10 and 6 floors to get to 16 and since 1 + 6 = 7, 7 is the answer to 16 - 9.

After using the paper model, students can mentally think of the elevator as they do subtraction problems. They might need to initially put the "elevator" numbers alongside a problem to determine the two parts of the difference. But eventually they can just quickly do the two parts mentally.

15 5	16 6	17 7	13 3
- 7 3	-9 1	-8 2	-5 5
8	7	9	8

The strategy can also be shown to students as two jumps on a number line, moving from the lower to the higher number, always with a stop at the number 10. (See page 31 for number line master).

13 - 8 = ?



5 + 3 = 8 so 13 - 5 = 8

Strategies for Learning Multiplication Facts

x 2 facts

- Skip counting
- Patterns

x 5 facts

- Skip counting
- Patterns
- halving - $\frac{1}{2}(10 \bullet n) = 5 \bullet n$

x 0 x 1 facts

- Compare to effects of 0 and 1 in addition and subtraction; post generalizations stated by students in their own words
- Create stories to access visual images of what 0 and 1 do

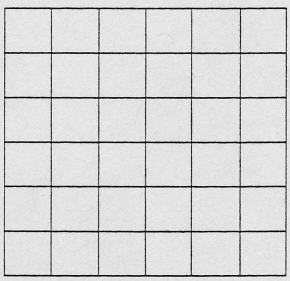
x 9 facts

- Nines on hands (see page 15)
- Patterns - tens place is 1 less than non-9 factor; digits in product equal 9 (see page 14). This is based on the 9 being one less than 10 and the distributive property.

$$9 \times n = y$$
$$(10-1) \times n = y$$
$$10n - n = y$$

Squares facts

- Use arrays to show the facts – when two factors are the same, their arrays are squares.



6 x 6 = 36

x 3 x 4 facts

- Arrays

- Work from known facts
  - 3s as a 2s fact plus another group 3 x 7 = (2 x 7) + (1 x 7)
  - 4s as two 2s (a "double double") 4 x 7 = (2 x 7) + (2 x 7)

x 6 x 7 x 8 facts

- Arrays

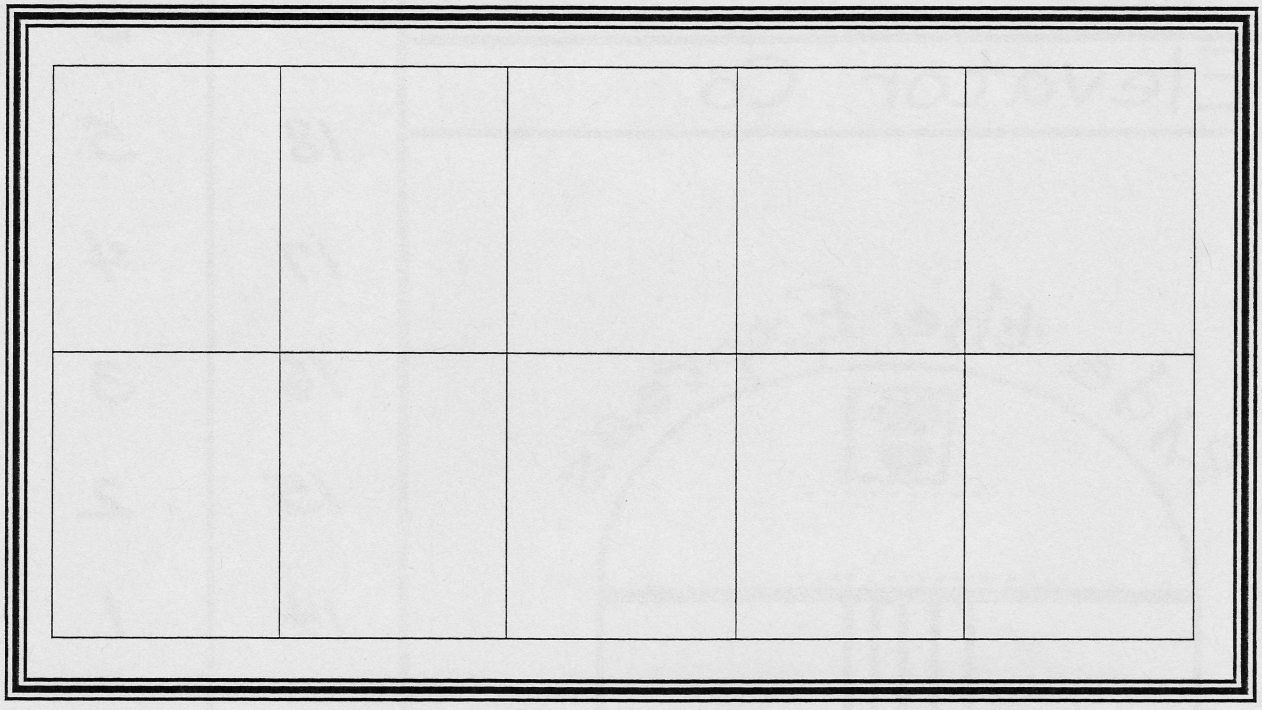
- Work from known facts
  - Think of half of the set, double it 6 x 7 = 2 (3 x 7)
  - Work from the 5s facts 6 x 7 = (5 x 7) + (1 x 7)

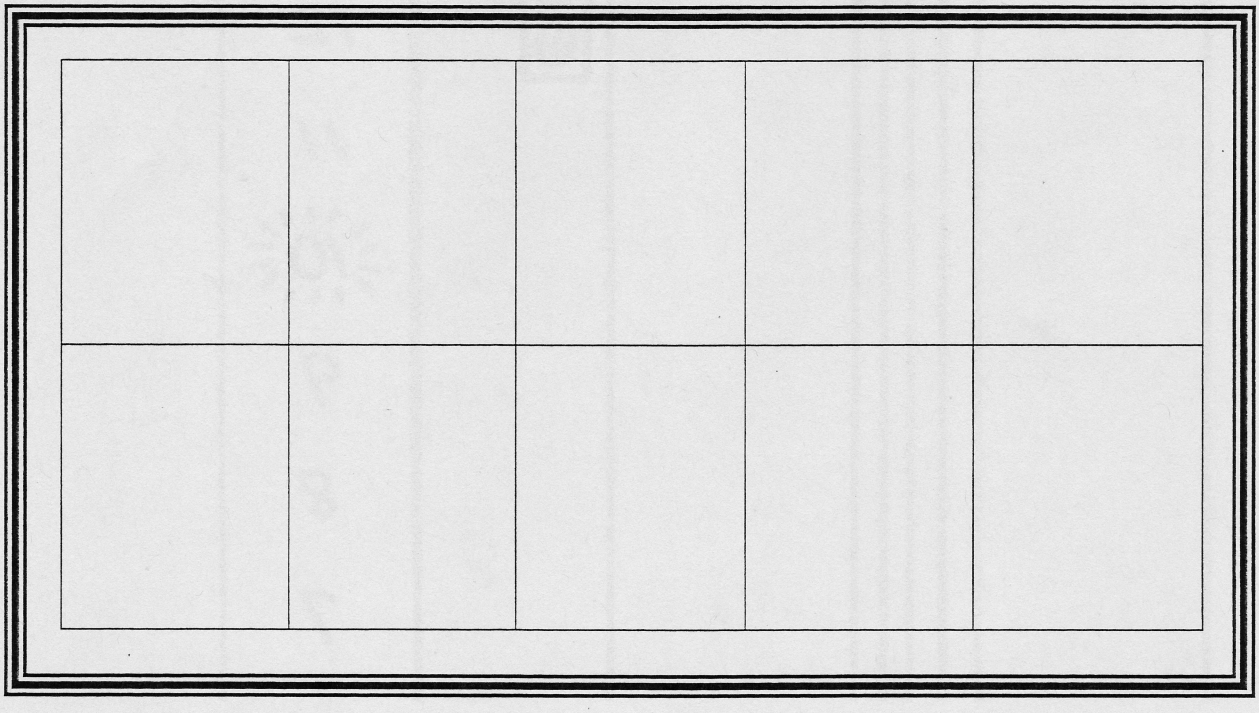
Strategies for Learning Division Facts

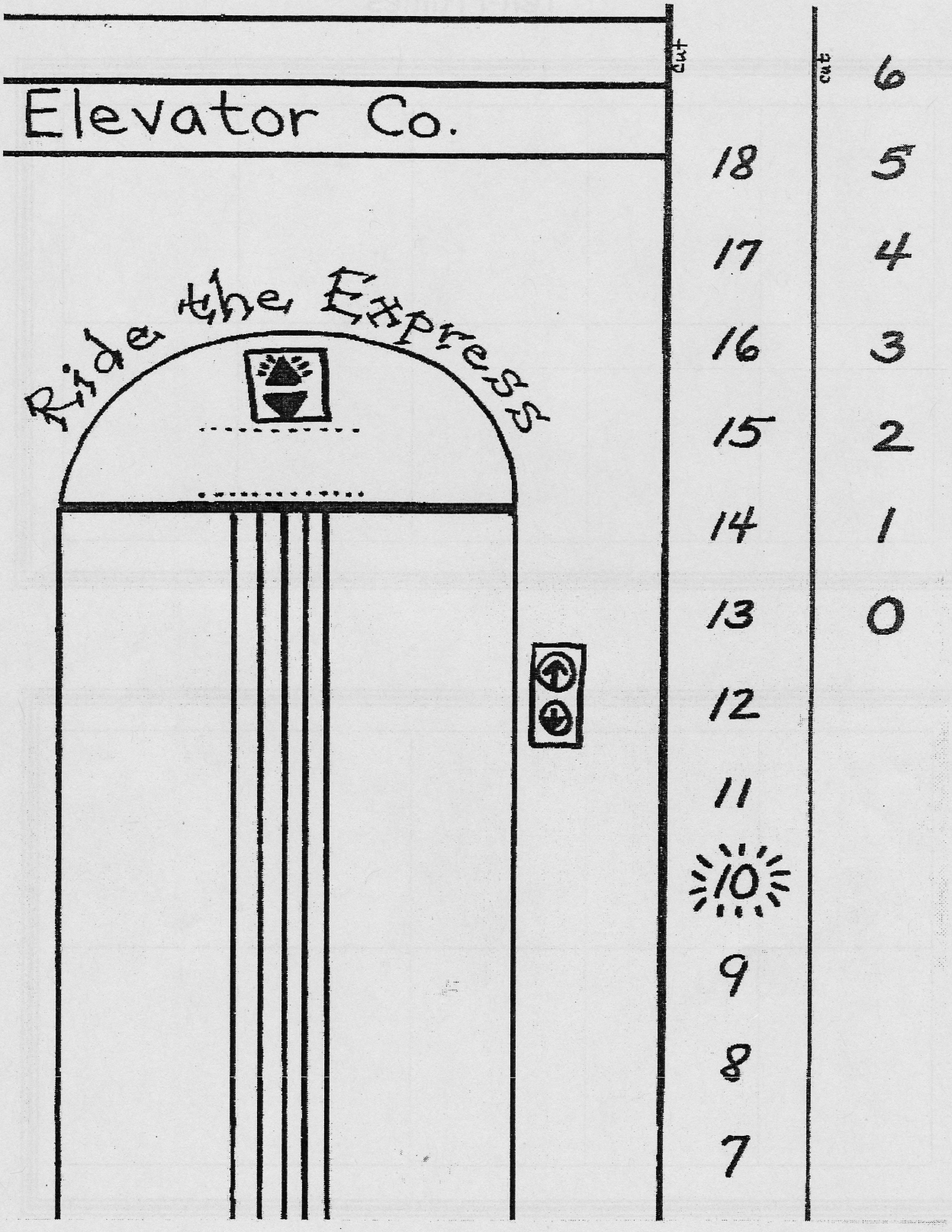
- Think multiplication – most students do division by thinking multiplication

- Work from a known fact

Ten-Frames






Elevator Co.
Ride the Express
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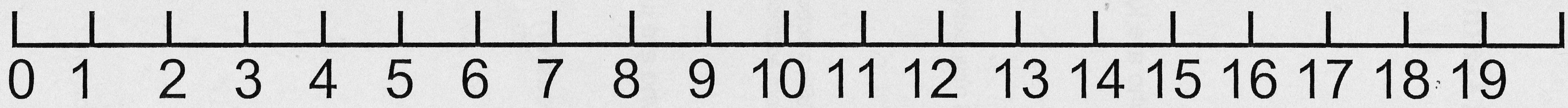

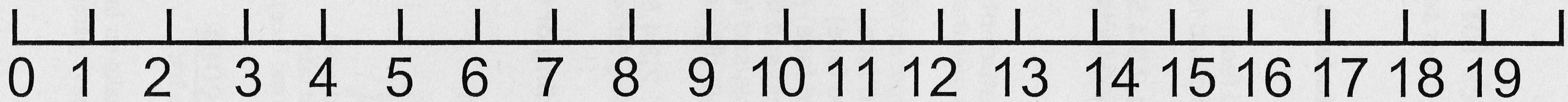

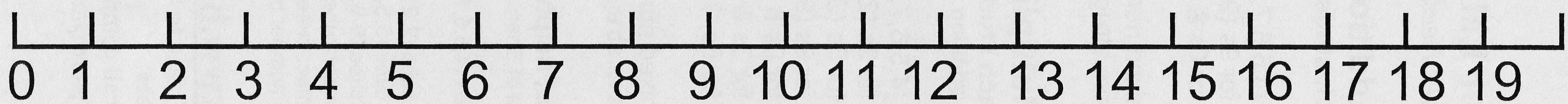

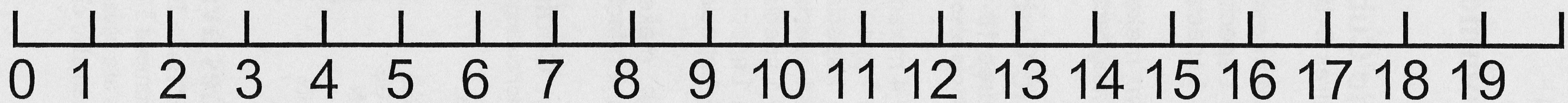

Recommendations for Labeling Cubes for Basic Fact Games

(see also "Cubes Needed" chart in games section, p. 36)

Basic Cube Set for Addition

Requires 6-10 cubes for each student or pair of students

Make 4 mixed number cubes (4,5,6,7,8,9)
Make 1 special number cube for 9s (9,9,9,9,9,pig/sad face)
Make 1 special number cube for 0s & 1s (0,0,1,1,1, pig/sad face)

Option: make 4 lower level mixed number cubes (0,1,2,3,4,5) for students who are just beginning fact work or who have struggled with learning facts

Basic Cube Set for Multiplication

Requires 10-14 cubes for each student or pair of students
If students also have cubes from addition facts (see above) then they will only need 4 new cubes

Make 4 mixed number cubes (4,5,6,7,8,9) *(can reuse from addition cubes)*
Make 1 special number cube for 2s (2,2,2,2,2, pig/sad face)
Make 1 special number cube for 5s (5,5,5,5,5, pig/sad face)
Make 1 special number cube for 9s (9,9,9,9,9, pig/sad face) *(can reuse from addition cubes)*
Make 1 special number cube for 0s & 1s (0,0,1,1,1, pig/sad face) *(can reuse from addition)*
Make 1 special number cube for 3s & 4s (3,3,4,4,4, pig/sad face)
Make 1 special number cube for 6s, 7s & 8s (6,7,7,8,8, pig/sad face)

Option: make 4 lower level mixed number cubes (0,1,2,3,4,5) for students who are just beginning fact work or who have struggled with learning facts

Color Coding (optional but helpful – makes finding or sorting cubes easier)

For color coding use different colors of Sharpie® pens.

BLACK	mixed number cubes (4,5,6,7,8,9, or 0,1,2,3,4,5)
PURPLE	2s special number cube (2,2,2,2,2,pig/sad face)
ORANGE	5s special number cube (5,5,5,5,5 pig/sad face)
GREEN	9s special number cube (9,9,9,9,9 pig/sad face)
RED	0s & 1s special number cube (0,0,1,1,1, pig/sad face)
BROWN	3s & 4s special number cube (3,3,4,4,4, pig/sad face)
BLUE	6s, 7s & 8s special number cube (6,7,7,8,8 pig/sad face)

Cubes available at www.bayerwood.com

(go to "Cubes" for price list)
Recommend 3/4", 7/8" or 1" cubes.
Cubes are usually 5¢-9¢ each – call company for price breaks on large orders.
You can also check a local craft store (may be pricey) or have parents make cubes for you.